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FAULT DIAGNOSIS

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Final Report ONR Contract N00014-79-C-0170

Fault Diagnosis

R. Saeks, Principal Investigator

Summary

The primary purpose of this contract was to assist the Naval Ocean System Center in implementing a fault diagnosis algorithm previously developed under ONR contract by the principle investigator. To this end our major activity was directed towards the development of an efficient data base for the symbolic transfer function data required by the algorithm. The resultant data base is described in detail in this report and is characterized by both reduced storage requirements and reduced retreival time as compared to previous approaches. By combining this data base with the ATE and interface work carred out by NOSC we believe that a viable fault diagnosis algorithm for linear analog circuits can be implemented. The remaining step is to actually encode a software package which incorporated these ideas.

I. Introduction

Historically, symbolic network analysis has been motivated by the problems of circuit design and, as such, the emphasis has been placed on quickly and efficiently obtaining symbolic transfer function from a given set of circuit specifications.[2,3] In an operational or maintenance environment, however, one is typically given a prescribed nominal circuit and desires determine the effect of various (possibly large) perturbations thereon. This is the case in a power system where one is given a fixed network and desires to determine the effect of proposed modifications thereto. Alternatively, in the problem of analog circuit fault diagnosis one desires to simulate the effect of a number of alternative failures to compare the simulated data with the observed failure data.[4]

In such an operational or maintenance environment numerous perturbations of the nominal circuit are studied and, as such, significant computational efficiencies can be obtained if one first generates a data base in terms of the nominal circuit parameters and then extracts the appropriate symbolic transfer function from the data base each time a different symbolic transfer is required. Of course the benefit to be achieved via such an approach is dependent on the size of the data base and the ease with which a symbolic transfer function may be retrieved therefrom.

The obvious manner in which to generate such a data base is to simply pre-compute the coefficients of all required symbolic transfer functions and store them in the data base. Retreival from such a data base is, of course, immediate but the data base may become overly large. Indeed, the number of transfer functions which must be stored is $O(\underline{k}^{\underline{p}})$ where $\underline{k}$ is the total number of potentially variable circuit parameters and $\underline{p}$ is the maximum number of circuit parameters which may vary simultaneously. An alternative approach is to store the nominal transfer function information and then use Householder's formula[1] to compute the required symbolic transfer functions. In such a data base we need only store $O(\underline{n}^2)$ transfer functions where $\underline{n}$ is the total number of component output terminals but retreival requires

$O(\underline{n}^3+p^3)$ multiplications where p is the actual number of circuit parameters which vary simultaneously. Since, in practice, $\underline{n} \gg p$ the retreival process requires approximately $O(\underline{n}^3)$ multiplications and is dominated by the large dimensional matrix multiplication required by Householder's formula rather than the low dimensional inverse.

In the present report we will formulate an alternative data base for the symbolic transfer functions which also requires $O(\underline{n}^2)$ entries, but for which retreival requires only $O(p^3)$ multiplications. Since p is typically small this is tantamount to immediate retreival.

In the remainder of this introduction we will review the properties of the component connection model for a large scale circuit or system[1] which serves as the starting point for our theory. The data base and retreival formulae for the case where $p \leq 2$ are formulated in section 2. while the general retreival formula is derived in section 3. Section 4. is devoted to the problem of retreiving sensitivity formulae from the data base while section 5. deals with the problem of updating the data base when the nominal circuit parameters are changed. Finally, section 6. is devoted to an example illustrating the theory.

The component connection model is an algebraic model for an interconnected dynamical system which subsumes the classical topological models but is more readily manipulated both analytically and computationally. The motivation and justification of the model are discussed in detail in reference 1 and will not be repeated here. The component connection model takes the form of the set of simultaneous equations

$$b = Z(j\omega)a \tag{1.1}$$

$$a = L_{11}b + L_{12}u \tag{1.2}$$

and

$$y = L_{21}b + L_{22}u \tag{1.3}$$

Here, Z (=Z(jω)) is a frequency dependent matrix characterizing the decoupled system components with composite component input and output vectors a and b, respectively. On the other hand the L_{ij}; i,j = 1,2; matrices are frequency independent connection matrices characterizing the coupling between the composite component vectors, a and b, and the composite system input and output vectors, u and y, respectively.

A little algebra with the component connection equations will readily reveal that

$$S = L_{22} + L_{21}(1 - ZL_{11})^{-1}ZL_{12} \tag{1.4}$$

where S (= S(jω)) is the composite system transfer function matrix[1] characterizing the external behavior of the system via

$$y = S(j\omega)u \tag{1.5}$$

Often, rather than working with the entire S matrix we find it convenient to work with its individual entries; s^{qv}, q = 1,2, ... ,$\underline{q}$ and v = 1,2, ...,$\underline{v}$: which are related to the component connection model via

$$s^{qv} = L_{22}^{qv} + L_{21}^{q}(1 - ZL_{11})^{-1}ZL_{12}^{v} \tag{1.6}$$

Here L_{22}^{qv} is the q-v entry in L_{22}; q = 1,2, ... ,$\underline{q}$ and v = 1,2, ...,$\underline{v}$; L_{21}^{q} is the qth row of L_{21}; q = 1,2, ... ,$\underline{q}$; and L_{12}^{v} is the vth column of L_{12}; v = 1,2, ... , $\underline{v}$.

Finally, since we are interested in analyzing the effects of perturbing one or more components from their nominal values, we decompose Z into nominal and perturbation terms in the form

$$Z = Z_o + Z_1 \tag{1.7}$$

where

$$Z_1 = \sum_{k=1}^{p} c^k \delta^k r^k \tag{1.8}$$

Here, c^k (= $c^k(j\omega)$) is a column vector, r^k (= $r^k(j\omega)$) is a row vector, and δ^k is the scalar perturbation for the kth potentially variable component parameter. In a typical application one is given c^k, r^k, and δ^k ; $k = 1,2, \ldots ,\underline{k}$; characterizing $\underline{k}$ potentially variable component parameters though at most $\underline{p}$ such parameters vary in any given analysis; $p \le \underline{p} \le \underline{k}$. Indeed, $p \ll \underline{k}$ in most applications. Finally, we note that Z_1 can be expressed more concisely in the form

$$Z = C\Delta R \tag{1.9}$$

where

$$C = [c^1 \mid c^2 \mid \ldots \mid c^p] \tag{1.10}$$

$$R = \begin{bmatrix} r^1 \\ \hline r^2 \\ \hline \vdots \\ \hline r^p \end{bmatrix} \tag{1.11}$$

and

$$\Delta = \begin{bmatrix} \delta^1 & & & \\ & \delta^2 & & \\ & & \ddots & \\ & & & \delta^p \end{bmatrix} \tag{1.12}$$

The above described notation formulated for the component connection model is summarized in table 1.

Matrix	Type	Dimension	Index
a	composite component input vector	$\underline{m} \times 1$	-
b	composite component output vector	$\underline{n} \times 1$	-
u	composite system input vector	$\underline{v} \times 1$	-
y	composite system output vector	$\underline{q} \times 1$	-
L_{11}	connection matrix	$\underline{m} \times \underline{n}$	-
L_{21}	connection matrix	$\underline{q} \times \underline{n}$	-
L_{21}^{q}	qth row of L_{21}	$\underline{1} \times \underline{n}$	$q = 1,2, \ldots ,\underline{q}$
L_{12}	connection matrix	$\underline{m} \times \underline{v}$	-
L_{12}^{v}	vth column of L_{12}	$\underline{m} \times 1$	$v = 1,2, \ldots ,\underline{v}$
L_{22}	connection matrix	$\underline{q} \times \underline{v}$	-
L_{22}^{qv}	q-v entry in L_{22}	1×1	$q =1,2, \ldots ,\underline{q}$; $v=1,2, \ldots ,\underline{v}$
S	composite system transfer function matrix	$\underline{q} \times \underline{v}$	-
S^{qv}	q-v entry in S	1×1	$q=1,2, \ldots ,\underline{q}$; $v=1,2, \ldots ,\underline{v}$
Z	composite component transfer	$\underline{n} \times \underline{m}$	-
Z_o	nominal composite component	$\underline{n} \times \underline{m}$	-
Z_1	composite component transfer function perturbation matrix	$\underline{n} \times \underline{m}$	-
c^k	column vector characterizing perturbation of kth parameter	$\underline{n} \times 1$	$k = 1,2, \ldots ,\underline{k}$
C	array of the c^k vectors for the parameters chich actual vary (row[c^k])	$\underline{n} \times p$	-
r^k	row vector characterizing	$1 \times \underline{m}$	$k = 1,2, \ldots ,k$
R	array of r^k vectors for the parameters which actually vary (col[r^k])	$p \times \underline{m}$	-
δ^k	kth variable parameter	1×1	$k = 1,2, \ldots ,\underline{k}$
Δ	array of δ^k's for parameters which actually vary (diag[δ^k])	$p \times p$	-

Table 1. Summary of Component Connection Model

II. The Data Base

Our data base is composed of the following family of (frequency dependent) scalar transfer functions

$$s_o^{qv} = L_{22}^{qv} + L_{21}^{q}(1 - Z_o L_{11})^{-1} Z_o L_{21}^{v} \quad ; \quad q = 1,2, \ldots, \underline{q}; \; v = 1,2, \ldots, \underline{v} \qquad 2.1$$

$$b^{qj} = L_{21}^{q}(1 - Z_o L_{11})^{-1} c^j \quad ; \quad q = 1,2, \ldots, \underline{q}; \; j = 1,2, \ldots, \underline{k} \qquad 2.2$$

$$d^{kv} = r^k [1 + L_{11}(1 - Z_o L_{11})^{-1} Z_o] L_{12}^{v}; \quad k = 1,2, \ldots, \underline{k} \quad v = 1,2, \ldots, \underline{v} \qquad 2.3$$

and

$$e^{kj} = r^k L_{11}(1 - Z_o L_{11})^{-1} c^j \quad ; \quad k,j = 1,2, \ldots, \underline{k} \qquad 2.4$$

Here, $\underline{q}$ and $\underline{v}$ denote the number of external system inputs and outputs which are typically few in number. As such, the e^{kj} array composed of $\underline{k}^2$ entries dominates the data base. Also note that all of the entries in the data base are formulated in terms of the nominal component values and, as such, the data base may be generated off-line without a priori knowledge of the perturbations to be analyzed. Finally, the entire data base may be generated with the aid of only a single $\underline{n}$ by $\underline{n}$ (sparse) matrix inverse.

Now, if we assume that only a single parameter is perturbed, i.e.

$$Z_1 = c^k \delta^k r^k \qquad 2.5$$

for some fixed $k = 1,2, \ldots, \underline{k}$, to retrieve s^{qv} from the data base we must evaluate

$$s^{qv} = L_{22}^{qv} + L_{21}^{q}(1 - [Z_o + c^k \delta^k r^k] L_{11})^{-1} [Z_o + c^k \delta^k r^k] L_{12}^{v} \qquad 2.6$$

in terms of the elements of our data base and the variable parameter, δ^k. To this end we invoke Householder's formula[1]

$$(W + XY)^{-1} = W^{-1} - W^{-1}X(1 + YW^{-1}X)^{-1}YW^{-1} \qquad 2.7$$

with $W = (1 - Z_oL_{11})$, $X = -c^k\delta^k$, and $Y = r^kL_{11}$ obtaining

$$(1 - [Z_o + c^k\delta^k r^k]L_{11})^{-1} = [(1 - Z_oL_{11}) + (-c^k\delta^k)(r^kL_{11})]^{-1}$$

$$= (1-Z_oL_{11})^{-1} + (1-Z_oL_{11})^{-1}c^k\delta^k(1-4^kL_{11}(1-Z_oL_{11})^{-1}c^k\delta^k)^{-1}r^kL_{11}(1-Z_oL_{11})^{-1}$$

$$= (1-Z_oL_{11})^{-1} = \frac{(1-Z_oL_{11})^{-1}c^k\delta^k r^kL_{11}(1-Z_oL_{11})^{-1}}{1 - \delta^k e^{kk}} \qquad 2.8$$

Now, upon substitution of 2.8 into 2.6 we obtain

$$s^{qv} = L^{qv}_{22} + L^q_{21}(1-[Z_o+c^k\delta^k r^k]L_{11})^{-1}[Z_o+c^k\delta^k r^k]L^v_{12}$$

$$= L^{qv}_{22} + L^q_{21}(1-Z_oL_{11})^{-1}[Z_o+c^k\delta^k r^k]L^v_{12}$$

$$+ \frac{L^q_{21}(1-Z_oL_{11})^{-1}c^k\delta^k r^kL_{11}(1-Z_oL_{11})^{-1}[Z_o+c^k\delta^k r^k]L^v_{12}}{1 - \delta^k e^{kk}}$$

$$= s^{qv}_o + \delta^k b^{qk} r^k L^v_{12} + \frac{\delta^k b^{qk} r^k L_{11}(1-Z_oL_{11})^{-1}Z_oL^v_{12} + (\delta^k)^2 b^{qk} e^{kk} r^k L^v_{12}}{1 - \delta^k e^{kk}}$$

$$= s^{qv}_o + \frac{\delta^k b^{qk} d^{kv} + (\delta^k)^2[-b^{qk}e^{kk}r^kL^v_{12} + b^{qk}e^{kk}r^kL^v_{12}]}{1 - \delta^k e^{kk}} = s^{qv}_o + \frac{\delta^k b^{qk} d^{kv}}{1 - \delta^k e^{kk}} \qquad 2.9$$

which is the desired symbolic transfer function.

If we assume that two parameters are perturbed; that is

$$Z_1 = c^k\delta^k r^k + c^j\delta^j r^j \qquad 2.10$$

a similar formula can be obtained wherein Householder's formula is applied twice. Since this formula is subsumed by the general retreival formula derived in the following section, we simply state the result without proof. In particular,

$$s^{qv} = L_{22}^{qv} + L_{21}^{q}(1 - [Z_o+c^k\delta^k r^k + c^j\delta^j r^j]L_{11})^{-1}[Z_o+c^k\delta^k r^k+c^j\delta^j r^j]L_{12}^{v}$$

$$= s_o^{qv} + \frac{\delta^k b^{qk} d^{kv}+\delta^j b^{qj} d^{jv}+\delta^k\delta^j(-e^{kk} b^{qj} d^{jv}-e^{jj} b^{qk} d^{kv}+e^{kj} b^{qj} d^{jv}+e^{jk} b^{qj} d^{kv})}{1 - \delta^k e^{kk} - \delta^j e^{jj} + \delta^k\delta^j(e^{kk}e^{jj} - e^{kj}e^{jk})} \qquad 2.11$$

III. Retreival Theorem

As is apparent from equation 2.11, our retreival formulas are quite complex, even for the case p = 2 and, as such, a more compact notation is required if they are to be tractable. To this end, we assume that δ^k; k = 1,2, ... ,p: denote the potentially variable parameters and that

$$Z_1 = \sum_{k=1}^{p} c^k \delta^k r^k = C \Delta R \qquad 3.1$$

Of course, the same expression applies to any set of p potentially variable parameters given an appropriate change of the index set. To obtain the required symbolic transfer function for

$$S = L_{22}+L_{21}(1-[Z_o+Z_1]L_{11})^{-1}[Z_o+Z_1]L_{12} \qquad 3.2$$

with the above specified Z_1 we now define the following matrices made up of elements from our data base

$$S_o = \begin{bmatrix} s_o^{11} & s_o^{12} & \cdots & s_o^{1\underline{v}} \\ s_o^{21} & s_o^{22} & \cdots & s^{2\underline{v}} \\ \vdots & \vdots & & \vdots \\ s_o^{q1} & s_o^{q2} & \cdots & s_o^{q\underline{v}} \end{bmatrix} \qquad 3.3$$

$$B = \begin{bmatrix} b^{11} & b^{12} & \cdots & b^{1p} \\ b^{21} & b^{22} & \cdots & b^{2p} \\ \vdots & \vdots & & \vdots \\ b^{q1} & b^{q2} & \cdots & b^{qp} \end{bmatrix} \qquad 3.4$$

$$D = \begin{bmatrix} d^{11} & d^{12} & \dots & d^{1\underline{v}} \\ d^{21} & d^{22} & \dots & d^{2\underline{v}} \\ \vdots & \vdots & & \vdots \\ d^{p1} & d^{p2} & & d^{p\underline{v}} \end{bmatrix} \qquad 3.5$$

$$E = \begin{bmatrix} e^{11} & e^{12} & \dots & e^{1p} \\ e^{21} & e^{22} & \dots & e^{2p} \\ \vdots & \vdots & & \vdots \\ e^{p1} & e^{p2} & \dots & e^{pp} \end{bmatrix} \qquad 3.6$$

while Δ is defined as per equation 1.12.

<u>THEOREM</u>: Using the above notation

$$S = L_{22}+L_{21}(1-[Z_oZ_1]L_{11})^{-1}[Z_o+Z_1]L_{12} = S_o + B(1 - \Delta E)^{-1}\Delta D$$

Proof: First, we observe that

$$S_o = L_{22} + L_{21}(1-Z_oL_{11})^{-1}Z_oL_{12} \qquad 3.7$$

is just the nominal system transfer function matrix while

$$B = L_{21}(1-Z_oL_{11})^{-1}C \qquad 3.8$$

and

$$D = R[1 + L_{11}(1-Z_oL_{11})^{-1}Z_o]L_{12} = R(1-L_{11}Z_o)^{-1} \qquad 3.9$$

via Householder's formula. Finally,

$$E = RL_{11}(1-Z_oL_{11})^{-1}C$$

where R and C are as defined by equations 1.10 and 1.11. As such,

$$(1 - \Delta E)^{-1} = (1 - \Delta RL_{11}(1-Z_oL_{11})^{-1}C)^{-1}$$

$$= [1 + \Delta RL_{11}(1 - (1-Z_oL_{11})^{-1}C\Delta RL_{11})^{-1}(1-Z_oL_{11})^{-1}C]$$

$$= [1 + \Delta RL_{11}(1 - Z_oL_{11} - Z_1L_{11})^{-1}C]$$

$$= [1 + \Delta RL_{11}(1-ZL_{11})^{-1}C] \qquad 3.10$$

where we have invoked Householder's formula with $Z = 1$, $X = \Delta RL_{11}$, and $Y = (1-Z_oL_{11})^{-1}C$; and equation 1.9. As such,

$$S_o + B(1 - \Delta E)^{-1}\Delta D = S_o + L_{21}(1-Z_oL_{11})^{-1}C[1 + \Delta RL_{11}(1-ZL_{11})^{-1}C]\Delta R(1-L_{11}Z_o)^{-1}L_{12}$$

$$= S_o + L_{21}(1-Z_oL_{11})^{-1}[Z_1 + Z_1L_{11}(1-ZL_{11})^{-1}Z_1](1-L_{11}Z_o)^{-1}L_{12}$$

$$= S_o + L_{21}(1-Z_oL_{11})^{-1}\{[(1-ZL_{11})+Z_1L_{11}](1-ZL_{11})^{-1}\}Z_1(1-L_{11}Z_o)^{-1}L_{12}$$

$$= S_o + L_{21}(1-Z_oL_{11})^{-1}(1-Z_oL_{11})(1-ZL_{11})^{-1}Z_1(1-L_{11}Z_o)^{-1}L_{12}$$

$$= S_o + L_{21}(1-ZL_{11})^{-1}Z_1(1-L_{11}Z_o)^{-1}L_{12}$$

$$= L_{22} + L_{21}(1-Z_oL_{11})^{-1}Z_oL_{12} + L_{21}(1-ZL_{11})^{-1}Z_1(1-L_{11}Z_o)^{-1}L_{12}$$

$$= L_{22} + L_{21}Z_o(1-L_{11}Z_o)^{-1}L_{12} + L_{21}(1-ZL_{11})^{-1}Z_1(1-L_{11}Z_o)^{-1}L_{12}$$

$$= L_{22} + L_{21}[Z_o + (1-ZL_{11})^{-1}Z_1](1-L_{11}Z_o)^{-1}L_{12}$$

$$= L_{22} + L_{21}(1-ZL_{11})^{-1}[(1-ZL_{11})Z_o + Z_1](1-L_{11}Z_o)^{-1}L_{12}$$

$$= L_{22} + L_{21}(1-ZL_{11})^{-1}[Z - ZL_{11}Z_o](1-L_{11}Z_1)^{-1}L_{12}$$

$$= L_{22} + L_{21}(1-ZL_{11})^{-1}Z[1-L_{11}Z_o](1-L_{11}Z_o)^{-1}L_{12}$$

$$= L_{22} + L_{21}(1-ZL_{11})^{-1}ZL_{12} = S$$

as required. ////////

IV. Sensitivity Formulae

If one is working directly with the component connection model, it is well known[4] that the sensitivity of S with respect to a parameter, δ^i, can be computed via the formula

$$\left[\frac{dS}{d\delta^i}\right] = L_{21}(1-ZL_{11})^{-1}\left[\frac{dZ}{d\delta^i}\right][1 + L_{11}(1-ZL_{11})^{-1}Z]L_{12} \qquad 4.1$$

and hence it is appropriate to ask whether or not such a sensitivity matrix can also be computed from our data base. Since the expression

$$S = S_o + B(1 - \Delta E)^{-1}\Delta D \qquad 4.2$$

is formally identical to 1.4, if $1 \leq i \leq p$ we may write

$$\left[\frac{dS}{d\delta^i}\right] = B(1-\Delta E)^{-1}M_i[1 + E(1-\Delta E)^{-1}\Delta]D \qquad 4.3$$

where

$$M_i = \frac{d\Delta}{d\delta^i} = \begin{bmatrix} 0 & & & & & & \\ & 0 & & & & & \\ & & 0 & & & & \\ & & & \ddots & & & \\ & & & & 1 & & \\ & & & & & \ddots & \\ & & & & & & 0 \end{bmatrix} \qquad 4.4$$

with the one appearing in the ith diagonal entry. Clearly, the expression can be computed directly from the data base with the same level of computational effort as required for the retreival formula.

In the case where δ^i is not included in the given set of parameters which deviate from nominal, $i > p$ in our notation, we must first augment the B, E, D, and Δ matrices to include δ^i and then apply equation 4.3 to the augmented system. To this end we let

$$B^i = \begin{bmatrix} b^{11} & b^{12} & \dots & b^{1p} & b^{1i} \\ b^{21} & b^{22} & \dots & b^{2p} & b^{2i} \\ \vdots & \vdots & & \vdots & \vdots \\ b^{\underline{q}1} & b^{\underline{q}2} & & b^{\underline{q}p} & b^{\underline{q}i} \end{bmatrix} \qquad 4.5$$

$$D^i = \begin{bmatrix} d^{11} & d^{12} & \dots & d^{1\underline{v}} \\ d^{21} & d^{22} & \dots & d^{2\underline{v}} \\ \vdots & \vdots & & \vdots \\ d^{p1} & d^{p2} & \dots & d^{p\underline{v}} \\ d^{i1} & d^{i2} & \dots & d^{i\underline{v}} \end{bmatrix} \qquad 4.6$$

$$E^i = \begin{bmatrix} e^{11} & e^{12} & \dots & e^{1p} & e^{1i} \\ e^{21} & e^{22} & \dots & e^{2p} & e^{2i} \\ \vdots & \vdots & & \vdots & \vdots \\ e^{p1} & e^{p2} & \dots & e^{pp} & e^{pi} \\ e^{i1} & e^{i2} & \dots & e^{ip} & e^{ii} \end{bmatrix} \qquad 4.7$$

and

$$\Delta^a = \left[\begin{array}{c|c} \Delta & 0 \\ \hline 0 & 0 \end{array}\right] \qquad 4.8$$

The we obtain the retreival formulae

$$S = S_0 + B^i(1 - \Delta^a E^i)^{-1}\Delta^a D^i \qquad 4.9$$

and

$$\left[\frac{dS}{d\delta^i}\right] = B^i(1-\Delta^a E^i)^{-1}M_{p+1}[1+E^i(1-\Delta^a E^i)^{-1}\Delta^a]D^i \qquad 4.10$$

V. Updating the Data Base

In many applications one uses a data base such as that described above, as a design tool to aid in simulating the effects of various proposed modifications to the system. When such a modification is finally implemented it is then necessary to update the data base to reflect the new nominal parameter values

$$\tilde{Z}_o = Z_o + \sum_{k=1}^{p} c^k \delta^k r^k = Z_o + C\Delta R \qquad 5.1$$

with the aid of Householder's formula we may compute

$$(1-\tilde{Z}_o L_{11}) = [(1-Z_o L_{11}) - C\Delta RL_{11}]^{-1} = (1-Z_o L_{11})^{-1}$$

$$+ (1-Z_o L_{11})^{-1} C[1 - \Delta RL_{11}(1-Z_o L_{11})^{-1} C]^{-1} \Delta RL_{11}(1-Z_o L_{11})^{-1}$$

$$= (1-Z_o L_{11})^{-1} + (1-Z_o L_{11})^{-1} C(1-\Delta E)^{-1} \Delta RL_{11}(1-Z_o L_{11})^{-1} \qquad 5.2$$

which upon substitution into equation 2.4 yields

$$\tilde{e}^{kj} = e^{kj} + [e^{k1}\ e^{k2}\ \dots\ e^{kp}](1-\Delta E)^{-1}\Delta \begin{bmatrix} e^{1j} \\ e^{2j} \\ \vdots \\ e^{pj} \end{bmatrix} \qquad 5.3$$

Similarly,

$$\tilde{b}^{qj} = b^{qk} + [b^{q1}\ b^{q2}\ \dots\ b^{qp}](1-\Delta E)^{-1}\Delta \begin{bmatrix} e^{1j} \\ e^{2j} \\ \vdots \\ e^{pj} \end{bmatrix} \qquad 5.4$$

$$\tilde{d}^{kv} = d^{kv} + [e^{k1}\ e^{k2}\ \dots\ e^{kp}](1-\Delta E)^{-1}\Delta \begin{bmatrix} d^{1v} \\ d^{2v} \\ \vdots \\ d^{pv} \end{bmatrix} \qquad 5.5$$

and

$$\bar{s}_o^{qv} = s_o^{qv} + [b^{q1} \; b^{q2} \; \ldots \; b^{qp}](1-\Delta E)^{-1}\Delta \begin{bmatrix} d^{1v} \\ d^{2v} \\ \vdots \\ d^{pv} \end{bmatrix} \qquad 5.6$$

As such, the entries in our data base can be updated with a computational effort which is commensurate with that required by the retreival formula.

VI. Examples

Consider the simple RC op-amp circuit shown in figure 1. The component connection

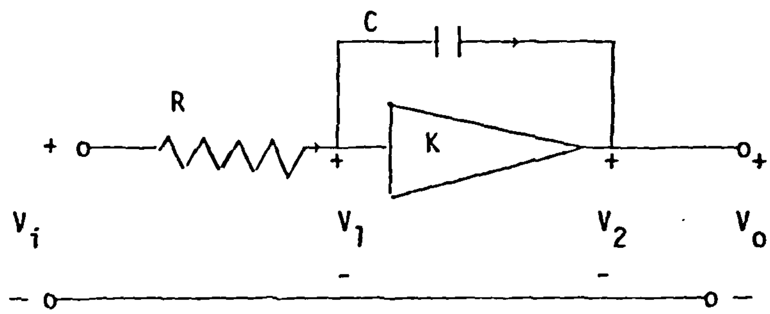

Figure 1: RC Op-amp circuit.

model for this circuit takes the form

$$\begin{bmatrix} i_c \\ v_r \\ v_2 \end{bmatrix} = \begin{bmatrix} sC & 0 & 0 \\ 0 & R & 0 \\ 0 & 0 & K \end{bmatrix} \begin{bmatrix} v_c \\ i_r \\ v_1 \end{bmatrix} \tag{6.1}$$

$$\begin{bmatrix} v_c \\ i_r \\ v_1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_c \\ v_r \\ v_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} v_i \end{bmatrix} \tag{6.2}$$

$$\begin{bmatrix} v_o \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_c \\ v_r \\ v_2 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} \begin{bmatrix} v_i \end{bmatrix} \tag{6.3}$$

Thus if all components taken to have nominal values of 1 we obtain

$$(1-Z_o L_{11}) = \begin{bmatrix} 1 & s & s \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \tag{6.4}$$

$$(1-Z_oL_{11})^{-1} = \begin{bmatrix} 1 & 0 & -s \\ 1 & 1 & -s \\ -1 & -1 & s+1 \end{bmatrix} \qquad 6.5$$

$$(1-Z_oL_{11})^{-1}Z_o = \begin{bmatrix} s & 0 & -s \\ s & 1 & -s \\ -s & -1 & s+1 \end{bmatrix} \qquad 6.6$$

$$L_{11}(1-Z_oL_{11})^{-1} = \begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & -s \\ -1 & -1 & s \end{bmatrix} \qquad 6.7$$

and

$$L_{11} + L_{11}(1-Z_oL_{11})^{-1}Z_o = \begin{bmatrix} 1 & 0 & -1 \\ s & 1 & -s \\ -s & -1 & s+1 \end{bmatrix} \qquad 6.8$$

Now, we may represent perturbations in the parameters C, R, and K via the matrices

$$c^1\delta^1r^1 = \begin{bmatrix} s \\ 0 \\ 0 \end{bmatrix} \delta^1 \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \qquad 6.9$$

$$c^2\delta^2r^2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \delta^2 \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \qquad 6.10$$

and

$$c^3\delta^3r^3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \delta^3 \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \qquad 6.11$$

Combining the appropriate c^k and r^j matrices with the above expressions as per equations 2.1 thru 2.4 we obtain the data base

$$s_o = 1 \qquad 6.12$$

$$b^1 = -s \quad b^2 = -1 \quad b^3 = s+1 \qquad 6.13$$

$$d^1 = 1 \quad d^2 = 0 \quad d^3 = 1 \qquad 6.14$$

and

$$e^{11} = 0 \quad e^{12} = 0 \quad e^{13} = -1$$

$$e^{21} = s \quad e^{22} = 0 \quad e^{23} = -s$$

$$e^{31} = -s \quad e^{32} = -1 \quad e^{33} = s \qquad 6.15$$

where we have deleted the q and v indices since we are dealing with a single-input single-output system.

Now, if one desires to compute the symbolic transfer function with respect to perturbations in the op-amp gain we have

$$S(s,\delta^3) = s_o + \frac{b^3\delta^3d^3}{1-\delta^3e^{33}} = \frac{1+\delta^3}{1-\delta^3s} \qquad 6.16$$

Recalling that δ^3 represents a perturbation from a nominal parameter value of $K_o = 1$ our actual gain is $K = K_o + \delta^3 = 1+\delta^3$, which upon substitution into 6.16 yields

$$S(s,K) = \frac{K}{(1-K)s + 1} \qquad 6.17$$

which is the classical gain formula for such a circuit.

Finally, if we desire to update our data base to reflect a new nominal value for the circuit parameters of C = 1, R = 1, and K = 2 we invoke equations 5.3 thru 5.6 with $\delta^3 = 1$ yielding

$$\tilde{s}_o = s_o + \left.\frac{b^3\delta^3 d^3}{1 - \delta^3 e^{33}}\right|_{\delta^3=1} = \frac{2}{1 - s} \tag{6.18}$$

$$\tilde{e}^{11} = e^{11} + \frac{e^{13}\delta^3 e^{31}}{1 - \delta^3 e^{33}} = 0 + \left.\frac{(-1)\delta^3(-s)}{1 - \delta^3(s)}\right|_{\delta^3 = 1} = \frac{s}{1 - s} \tag{6.19}$$

and similarly for the other elements of the data base.

VII. Conclusions

The preceeding development has been motivated by operational and maintenance considerations rather than the design considerations. In such an environment one typically deals with a fixed nominal system, but carries out repeated analyses thereon. As such, the cost of generating the required data base is secondary compared to the cost of storing the data base and retreiving information therefrom. In these respects we believe that our data base is near optimal. Since the number of system inputs and outputs is typically small our data base contains approximately $\underline{k}^2$ elements (actually $\underline{k}^2 + \underline{k}(\underline{v}+\underline{q}) + \underline{vq}$) where $\underline{k}$ is the total number of parameters which are potentially variable. This data base, however, contains sufficient information to permit one to retrieve symbolic transfer functions for any number $p \leq \underline{k}$ of variable parameters. Indeed, the number of variable parameters in a symbolic transfer function is reflected only in the cost of retreival which is on the order of p^3 multiplications (actually $p^3 + p^2\underline{v} + p\underline{v}(\underline{q}+1)$). Since p is typically small, say five or less, this is minimal.

VIII. References

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